Soluție

1. a) $\sum_{j=1}^{3}\left(a_{ij}+b_{ij}\right)=\sum_{j=1}^{3}a_{ij}+\sum_{j=1}^{3}b_{ij}=0,\ \forall i\in\{1,2,3\}$.

b) Dacă $A\in M\Rightarrow\det(A)=0.$

c) Fie $A=\left(a_{ij}\right)\in M$ şi $A^2=\left(c_{ij}\right)$ unde $c_{ij}=\sum_{k=1}^{3}a_{ik}a_{kj}$. Pentru fiecare $i\in\{1,2,3\}$ avem

$c_{i1}+c_{i2}+c_{i3}=\sum_{k=1}^{3}\left(a_{ik}a_{k1}+a_{ik}a_{k2}+a_{ik}a_{k3}\right)=\sum_{k=1}^{3}a_{ik}\left(a_{k1}+a_{k2}+a_{k3}\right)=0$.

2.a) $x_{1,2}=\pm\left(1+\sqrt{2}\right)\in\mathbb{Z}\left[\sqrt{2}\right]$.

b) $\mathbb{Z}\subset\mathbb{Z}\left[\sqrt{2}\right],\ \mathbb{Z}\subset\mathbb{Z}\left[\sqrt{3}\right]\Rightarrow\mathbb{Z}\subset\mathbb{Z}\left[\sqrt{2}\right]\cap\mathbb{Z}\left[\sqrt{3}\right]$. Fie
$x=a+b\sqrt{2}=c+d\sqrt{3}$ cu $a,b,c,d\in\mathbb{Z}\Rightarrow b\sqrt{2}-d\sqrt{3}\in\mathbb{Z}$ şi prin ridicare la pătrat rezultă $bd\sqrt{6}\in\mathbb{Z}\Rightarrow bd=0$.
Deci $b=0$ sau $d=0$, de unde $x\in\mathbb{Z}$.

c) Presupunem că există $f:\mathbb{Z}\sqrt{2}\to\mathbb{Z}\sqrt{3}$ morfism de inele. Cum $f(1)=1$ rezultă că $f(2)=f(1)+f(1)=2$.
Atunci $2=f\left(\sqrt{2}^{2}\right)=f^{2}\left(\sqrt{2}\right)$, deci $f\left(\sqrt{2}\right)=\pm\sqrt{2}\notin\mathbb{Z}\left[\sqrt{3}\right]$.